DAA-003-001513 Seat No. ________

Third Year B. Sc. (Sem. V) (CBCS) Examination

April / May – 2015

Mathematics : Paper - BSMT - 501 (A) (Theory)

(Mathematical Analysis - I & Group Theory)

Faculty Code : 003
Subject Code : 001513

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions : (1) This question paper contains two sections.
(2) All the questions are compulsory.
(3) Write answers of the MCQs in your answer-book only.
(4) Numbers written to the right indicate full marks of the question.

SECTION - I

1 Select the correct option from the given options (to answer the given questions) : **20**

(1) If P and P^* are two partitions of $[a, b]$ and $P \subset P^*$ then

(A) $\|P^*\| > \|P\|$ (B) $\|P^*\| \le \|P\|$

(C) $\|P^*\| \ge \|P\|$ (D) $\|P^*\| < \|P\|$

(2) $\int_{\underline{a}}^{b} f dx =$ ______

(A) lub $\{U(P, f)\}$ (B) lub $\{L(P, f)\}$

(C) glb $\{U(P, f)\}$ (D) glb $\{L(P, f)\}$

(3) If P is a partition of $[a, b]$ and if $n \to \infty$ then $\|P\| \to$ ____ .

(A) 1 (B) ∞

(C) 0 (D) -1

(4) $\underline{\int_{a}^{b}} f dx$ ____ $\overline{\int_{a}^{b}} f dx$

(A) $\leq$ (B) $<$

(C) $\geq$ (D) $>$

(5) $U(P, f)$ ____ $\overline{\int_{a}^{b}} f dx + \in$, where $\in > 0$.

(A) $>$ (B) $<$

(C) $\leq$ (D) $\geq$

(6) If (X, d) is a discrete metric space and $\delta = 1$ then

(A) $N(a, \delta) = \{\delta\}$ (B) $N(a, \delta) = \{a\}$

(C) $N(a, \delta) = \{X\}$ (D) $N(a, \delta) = X$

(7) Every finite subset of a metric space is ______.

(A) open

(B) closed

(C) neither open nor closed

(D) open and closed

(8) Cantor set is ______.

(A) open

(B) closed

(C) neither open nor closed

(D) open and closed

(9) Bod N = ______. (Boundary of N)

(A) N (B) R

(C) ϕ (D) N'

(10) For $a \in X$ and $\delta = 1, N(a, \delta)$ is _____.

(A) open

(B) closed

(C) neither open nor closed

(D) open and closed

(11) $(N, +)$ is a group ....

(A) True (B) False

(C) May or may not be (D) None of these

(12) $(Z_n, \bullet_n)$ is a group ....

(A) True, for every $n \in N$

(B) False, for every $n \in N$

(C) If n is a prime number

(D) None of these

(13) A non-empty subset H of a group G is a SUBGROUP of G iff ....

(A) $a * b^{-1} \in H \quad \forall a, b \in H$

(B) $a * b^{-1} \in G \quad \forall a, b \in H$

(C) $a^{-1} \in H$

(D) None of these

(14) A subgroup H of a group G is NORMAL of G if ....

(A) $Ha = aH$ for every $a \in G$

(B) $a * b \in G, \forall a, b \in H$

(C) $gHg^{-1} \in H$ for every $g \in G$

(D) None of these

(15) The permutation $\begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix} \in S_3$ is ....

(A) An odd permutation

(B) is a transposition

(C) A cyclic in S_3

(D) None of these

(16) The inverse of $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 5 & 4 & 1 & 3 \end{pmatrix} \in S_5$ is given by ....

(A) $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 4 & 2 & 5 & 1 & 3 \end{pmatrix} \in S_5$

(B) $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 5 & 4 & 2 & 1 \end{pmatrix} \in S_5$

(C) $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 4 & 1 & 5 & 3 & 2 \end{pmatrix} \in S_5$

(D) None of these

(17) If G is a group $a \in G$ then $O(a) = n$ then ....

(A) $O\left(a^q\right) > O(a);\ \forall q \in Z$

(B) $O\left(a^q\right) \geq O(a);\ \forall q \in Z$

(C) $O\left(a^q\right) \leq O(a);\ \forall q \in Z$

(D) None of these

(18) If for $a, b \in N$, $a * b = a^b$ then the operation $*$ is ....

(A) Commutative

(B) Not associative

(C) Not a binary operation

(D) None of these

(19) If U is a non-empty universal set and Δ is symmetric difference then for group $(P(U), \Delta)$ the identity element ....

(A) Does not exist

(B) is U

(C) is ϕ

(D) None of these

(20) Isomorphism of groups is ....

(A) Partial Order Relation

(B) Equivalence Relation

(C) Anti-symmetric Relation

(D) None of these.

SECTION - II

2 (a) Attempt any three : **6**

(1) Define :

(i) Neighbourhood

(ii) Interior point

(2) In usual notation prove that if $A \subset B$ then $A^\circ \subset B^\circ$.

(3) Define Cantor set.

(4) If $f(x) = \dfrac{20}{x}$ where $x \in [2, 20]$ then find $L(P, f)$ and $U(P, f)$ by taking partition $P = \{2, 4, 5, 20\}$.

(5) Define : Lower Riemann sum and Upper Riemann sum.

(6) Evaluate $\int_0^1 x^2 dx$ by using definition of R-integration.

(b) Attempt any **three** : **9**

(1) Let f be a bounded function defined on $[a, b]$. If P and P^* are two partition of $[a, b]$ such that $P \subset P^*$ then prove that

$$L(P, f) \le L(P^*, f) \le U(P^*, f) \le U(P, f)$$

(2) Evaluate : $\lim\limits_{n \to \infty} \left(\left(1 + \frac{1}{n}\right)\left(1 + \frac{2}{n}\right) \ldots\ldots\ldots \left(1 + \frac{4n}{n}\right) \right)^{1/n}$

(3) Prove that : $\dfrac{\pi^3}{51} \le \displaystyle\int_0^\pi \frac{x^2}{10 + 7\cos x} dx \le \frac{\pi^3}{9}$.

(4) If $E=\left\{1,\frac{1}{2},\frac{1}{3},\dots\dots\right\}$ then prove that E is neither open not closed subset of R.

(5) Prove that every closed interval $[a,b]$ is a closed set.

(6) In usual notation prove that A is closed $\Leftrightarrow \overline{A}=A$.

(c) Attempt any **two** : **10**

(1) State and prove general form of first mean value theorem of integral calculus.

(2) State and prove necessary and sufficient condition for a bounded function defined on $[a,b]$ to be Riemann integrable over $[a,b]$.

(3) State and prove first mean value theorem.

(4) State and prove the necessary and sufficient condition for a subset of metric space to be an open set.

(5) In usual notation prove that $\overline{E}$ is a closed set.

3 (a) Attempt any **three** : **6**

(1) Define :

(i) Binary operation

(ii) Cyclic group

(2) Define :

(i) Isomorphism of groups

(ii) Normal subgroup

(3) Define :

(i) Cycle

(ii) Alternating subgroup

(4) If $G = (\mathbb{Z}; +)$ and $H = 4\mathbb{Z}$ then write all the elements (cosets) of $G - H$.

(5) Define :

(i) Inner automorphism

(ii) Quotient Group.

(6) Prove that intersection of two subgroup is also a subgroup.

(b) Attempt any **three** : **9**

(1) If $G = R - \{-1\}$ and let * be defined as $a * b = a + b + ab$ for every $a, b, c \in G$ then prove that $(G, *)$ is a group.

(2) Prove that, if G is a group and if $a \in G$ is of order n then $a^m = e,$ for some integer m iff n/m.

(3) State and prove necessary and sufficient condition for subgroups.

(4) If H and K are normal subgroups of a group G with $H \cap K = \{e\}$ then prove that $kh = hk$ for each $h \in H$ and for each $k \in K$.

(5) Prove that any two disjoint cycles in S_n are commutative.

(6) If G is a cyclic group of order 24 generated by a and $H = \langle a^6 \rangle$ then write all the elements of G/H, construct the group table of G/H and specify its identity element and inverse elements.

(c) Attempt any **two** : **10**

(1) State and prove Lagrange's theorem for finite groups.

(2) Prove that set A_n of all Even permutations is a subgroup of the group S_n and $O(A_n) = \frac{n!}{2}$.

(3) For a given element a of a group G, prove that the set $H = \{x \in G / xa = ax\}$ is a subgroup of G.

(4) Prove that the isomorphism between two groups is transitive relation.

(5) State and prove Cayley's theorem for groups.